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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 2, 2016/2017

### BMS1024 – MANAGERIAL STATISTICS

(All sections / Groups)

11 MARCH 2017  
2.30 p.m – 4.30 p.m  
(2 Hours)

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#### INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FOURTEEN (14)** printed pages with:  
**Section A:** Ten (10) multiple choice questions (20%)  
**Section B:** Three (3) structured questions (80%)
2. Answer **ALL** questions.
3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.
4. Statistical tables are attached at the end of the question paper.
5. Students are allowed to use non-programmable scientific calculators with no restrictions.

**SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)**

There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.

1. The data below represent the amount of grams of carbohydrates per serving of breakfast cereal:

11	15	23	29	19	22	21	20	15	25	17
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Determine the third-quartile for this data:

- A. 23 grams  
B. 20 grams  
C. 19.73 grams  
D. 15 grams
2. A task force at a university sampled 200 students after the midterm exam to investigate whether they went to the entertainment centers before exam or spend time for revision. The task force also investigates whether they did well or poorly on the midterm exam. The following result was obtained:

	Perform well on Midterm exam	Did Poorly on Midterm exam
Self-Revision for Exam	80	20
Went to Entertainment Centers	30	70

What is the probability that a randomly selected student did well on the midterm exam or went to the entertainment centers before the exam?

- A. 100/200  
B. 0.9  
C. 0.15  
D. 110/200
3. On average, a worker earns \$15 per hour at a plant in China and is told only 2.5% of all workers make a higher wage. If the wage is normally distributed with standard deviation of wage rates is \$5 per hour, find the hourly wage for the plant.
- A. \$15 per hour  
B. \$24.8 per hour  
C. \$5.2 per hour  
D. \$2.34 per hour

Continued...

4. The local police department must write, on average, 5 tickets a day to keep department revenues at budgeted levels. Suppose the number of tickets written per day follows a Poisson distribution with a mean of 6.5 tickets per day. Interpret the value of the mean.
- A. Half of the days have more than 6.5 tickets written.
  - B. The number of tickets that is written most often is 6.5 tickets per day.
  - C. The mean has no interpretation since 0.5 ticket can never be written.
  - D. If we sampled all days, the expected number of tickets written would be approximately 7 tickets per day.
5. An economist is interested in studying the incomes of consumers in a particular country. The population standard deviation is known to be \$1,000. A random sample of 50 individuals resulted in a mean income of \$15,000. What is the upper end point in a 99% confidence interval for the average income?
- A. \$15,364
  - B. \$14,636
  - C. \$15,000
  - D. \$15,052
6. Health care issues are receiving much attention in both academic and political arenas. A sociologist recently conducted a survey of citizens over 60 years of age whose net worth is too high to qualify for Medicaid. Calculate the mean age of the senior citizens as following:
- |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|
| 60.60 | 71.35 | 82.30 | 63.56 | 65.56 | 75.40 |
|-------|-------|-------|-------|-------|-------|
- A. 60.60 years of age
  - B. 71.35 years of age
  - C. 65.56 years of age
  - D. 69.80 years of age
7. The probability that house sales will increase in the next 6 months is estimated to be 0.25. The probability that the interest rates on housing loans will go up in the same period is estimated to be 0.74. The probability that house sales or interest rates will go up during the next 6 months is 0.89. The probability that house sales will increase **but** interest rates will not during the next 6 months is:
- A. 0.065
  - B. 0.15
  - C. 0.11
  - D. 0.75

Continued...

8. The Wall Street Journal recently ran an article indicated differences in perception of sexual harassment on the job between men and women. The article claimed that women perceived the problem to be much more prevalent than did men. One question asked both men and women was: "Do you think sexual harassment is a major problem in the American workplace?". There were 24% of the men compared to 62% of the women responded yes. There are 150 women and 200 men were interviewed. What conclusion should be reached?
- A. Using 1 percent level of significance, there is sufficient evidence to conclude that women perceive the problem of sexual harassment on the job as much more prevalent than men.
  - B. There is insufficient evidence to conclude with at least 99% confidence that women perceive the problem of sexual harassment on the job as much more prevalent than men.
  - C. There is no evidence of a significant difference between the mean and women in their perception.
  - D. More information is needed to draw any conclusions from the data set.
9. How many tissues should Clark Kent Corporation package of Kleenex contain? Researchers determined that 60 tissues are the mean number of tissues used during a cold. A random sample of 39 Kleenex users yielded the following data on the number of tissues used during a cold:  $\bar{x} = 52, s = 22$ . Suppose the alternative you wanted to test was  $H_1: \mu < 60$ . Select the correct rejection region for  $\alpha = 5\%$ .
- A. Reject  $H_0$  if  $t > 1.686$
  - B. Reject  $H_0$  if  $t < -1.686$
  - C. Reject  $H_0$  if  $t > -1.686$
  - D. Reject  $H_0$  if  $t < 1.686$
10. A large national bank charges local companies for using their services. A bank official reported the results of a regression analysis designed to predict the bank's charges for services rendered to local companies. An independent variable used to predict service charges is the company's sales revenue. The result of the regression analysis is provided as below:

$$\hat{Y} = -2,700 + 20X$$

Which of the following statement is **true** for the  $Y$ -intercept of the line?

- A. All companies will be charged at least \$2,700 by the bank.
- B. There is no practical interpretation since the sales revenue of \$0 is a nonsensical value.
- C. About 2700% of the observed service charges fall within \$2,700 of the least squares line.
- D. For every 1-dollar increase in sales revenue, the service charges are decreased by \$2,700.

**Continued...**

**SECTION B: STRUCTURED QUESTIONS (80 MARKS)**

There are **THREE** questions in this section. Candidates **MUST** answer **ALL THREE** questions.

**Question 1 (25 Marks)**

- (a) According to a public census, one-fourth of all business are owned by women. If we randomly selected 17 businesses,
- (i) what is the probability between 11 until 14 of them are owned by women?  
(4 marks)
  - (ii) determine the probability at least 15 businesses are owned by women.  
(4 marks)
  - (iii) calculate the expected number of women who owned a business.  
(3 marks)
  - (iv) find the standard deviation of this distribution.  
(3 marks)
- (b) The number of magazine subscriptions per household is represented by the following probability distribution:

Magazine subscriptions per household	1	2	3	4
Probability	0.35	0.08	0.05	0.04

- (i) Calculate the probability from one to three magazine subscriptions per household.  
(3 marks)
- (ii) Compute the mean of magazine subscriptions per household.  
(3 marks)
- (iii) Determine the variance and standard deviation for the above random variable.  
(5 marks)

**Continued...**

**Question 2 (25 Marks)**

- (a) The routes of postal deliverers are carefully planned so that each deliverer works between 7 and 7.5 hours per shift. In an experiment to examine the amount of time deliverers (hours) actually spend completing their shifts, a random sample of 15 postal deliverers was secretly timed. The data samples are given as below:

6.9	6.9	7.3	7.0	7.0	6.8	6.6	7.0
6.7	6.5	6.9	6.2	7.1	6.8	6.7	

- (i) Determine 95% confidence interval of the mean shift time for all postal deliverers.  
(7 marks)
- (ii) Is there enough evidence at the 10% significance level to conclude that postal workers are on average spending less than 7 hours daily doing postal delivery?  
(8 marks)
- (b) Surveys have been widely used by politicians around the world as a way of monitoring the opinions of the electorate. Six months ago, a survey was undertaken to determine the degree of support for a national party leader. 56% indicated that they would vote for this politician from a sample of 1,100 respondents. Another survey of 800 voters in this month, revealed that 46% support the leader.

At the 5 percent significance level, can we infer that the national leader's popularity has decreased?

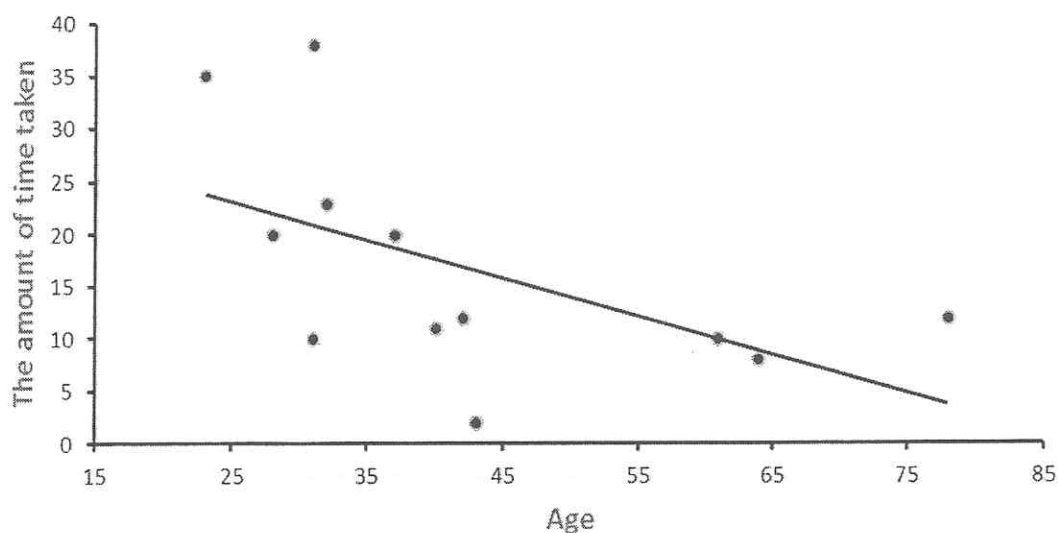
(10 marks)

Continued...

**Question 3 (30 Marks)**

- (a) In 2010, the United States conducted a study of the entire country. The study is completed by mailing questionnaires to the citizens. To ensure the questions are understandable, a random sample of Americans answer the questionnaire before it is sent out. As part of their analysis, they were recorded the amount of time (minutes) and ages (years) of the sample. These data, scatterplot and summary output of the relationship between the two variables are shown below:

Age (years)	Amount of time (minutes)
78	12
32	23
43	2
42	12
37	20
31	10
64	8
28	20
23	35
61	10
31	38
40	11



Continued...

**SUMMARY OUTPUT**

<i>Regression Statistics</i>	
Multiple R	0.1155
R Square	0.0133
Adjusted R Square	-0.0853
Standard Error	6.6781
Observations	12

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	6.0306	6.0306	0.1352	0.7207
Residual	10	445.9694	44.5969		
Total	11	452			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	10.8846	5.4755	1.9879	0.0749
age	-0.0443	0.1206	-0.3677	0.7207

- (i) State the dependent variable and the independent variable for the above regression model.  
(2 marks)
- (ii) Write the least square regression line for the above relationship between the two variables. State the unit of measurement for each variable.  
(4 marks)
- (iii) What do the coefficient of the regression line tell you about the relationship between respondent's age and the recorded time completed questionnaire?  
(3 marks)
- (iv) Determine the coefficient of correlation and discuss the role of this coefficient value for this model.  
(4 marks)
- (v) Predict the amount of time completing the questionnaire if the respondent is 18 years old. Is this estimation reliable? Explain.  
(4 marks)
- (vi) State the coefficient of determination and describe what it tells you.  
(3 marks)

Continued...



- (b) A USDA survey of grain production for selected areas in the United States yielded this information:

Product	2005		2015	
	Price (\$)	Quantity	Price (\$)	Quantity
Wheat	0.99	620	1.06	693
Corn	1.54	390	1.59	419
Oats	0.25	90	0.31	77
Rye	0.06	20	0.09	20

Compute and interpret the Laspeyres Price Index (LPI) and Paasche Price Index (PPI) for 2015 using 2005 as the base period.

**(10 marks)**

**End of Page**

## A. STATISTICAL FORMULAE

### A. DESCRIPTIVE STATISTICS

$$\text{Mean } (\bar{x}) = \frac{\sum_{i=1}^n X_i}{n}$$

$$\text{Standard Deviation } (s) = \sqrt{\frac{\sum_{i=1}^n X_i^2}{n-1} - \frac{(\sum_{i=1}^n X_i)^2}{n(n-1)}}$$

$$\text{Coefficient of Variation } (CV) = \frac{\sigma}{\bar{X}} \times 100$$

$$\text{Pearson's Coefficient of Skewness } (S_k) = \frac{3(\bar{X} - \text{Median})}{s}$$

### B. PROBABILITY

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B) \quad \text{if } A \text{ and } B \text{ are independent}$$

$$P(A | B) = P(A \text{ and } B) \div P(B)$$

#### Poisson Probability Distribution

If  $X$  follows a Poisson Distribution,  $P(\lambda)$  where  $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$   
then the mean =  $E(X) = \lambda$  and variance =  $VAR(X) = \lambda$

#### Binomial Probability Distribution

If  $X$  follows a Binomial Distribution  $B(n, p)$  where  $P(X=x) = {}^n C_x p^x q^{n-x}$   
then the mean =  $E(X) = np$  and variance =  $VAR(X) = npq$  where  $q = 1 - p$

#### Normal Distribution

If  $X$  follows a Normal distribution,  $N(\mu, \sigma)$  where  $E(X) = \mu$  and  $VAR(X) = \sigma^2$   
then  $Z = \frac{X - \mu}{\sigma}$

### C. EXPECTATION AND VARIANCE OPERATORS

$$E(X) = \sum [X \cdot P(X)]$$

$$VAR(X) = E(X^2) - [E(X)]^2 \quad \text{where } E(X^2) = \sum [X^2 \cdot P(X)]$$

$$\text{If } E(X) = \mu \text{ then } E(cX) = c\mu, \quad E(X_1 + X_2) = E(X_1) + E(X_2)$$

$$\text{If } VAR(X) = \sigma^2 \text{ then } VAR(cX) = c^2 \sigma^2,$$

$$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2 COV(X_1, X_2)$$

$$\text{where } COV(X_1, X_2) = E(X_1 X_2) - [E(X_1) E(X_2)]$$

### D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

$(100 - \alpha) \%$  Confidence Interval for Population Mean ( $\sigma$  Known) =  $\mu = \bar{X} \pm Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$

$(100 - \alpha) \%$  Confidence Interval for Population Mean ( $\sigma$  Unknown) =

$$\mu = \bar{X} \pm t_{\alpha/2, n-1} \left( \frac{s}{\sqrt{n}} \right)$$

$(100 - \alpha) \%$  Confidence Interval for Population Proportion =  $\hat{p} \pm Z_{\alpha/2} \sigma_{\hat{p}}$

Where  $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean =  $n \geq \left[ \frac{(Z_{\alpha/2})\sigma}{E} \right]^2$

Sample Size Determination for Population Proportion =  $n \geq \frac{(Z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

### E. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation ( $\sigma$ ) Known	Standard Deviation ( $\sigma$ ) Not Known
$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$
One Sample Proportion Test	
$z = \frac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$	
Two Sample Mean Test	
Standard Deviation ( $\sigma$ ) Known	Standard Deviation ( $\sigma$ ) Not Known
$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ where $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$
Two Sample Proportion Test	
$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ where $p = \frac{X_1 + X_2}{n_1 + n_2}$	
where $X_1$ and $X_2$ are the number of successes from each population	

**F. REGRESSION ANALYSIS****Simple Linear Regression****Population Model:**  $Y = \beta_0 + \beta_1 X_1 + \varepsilon$ **Sample Model:**  $\hat{y} = b_0 + b_1 x_1 + e$ **Correlation Coefficient**

$$r = \frac{\sum XY - \left[ \frac{\sum X \sum Y}{n} \right]}{\sqrt{\left[ \sum X^2 - \left( \frac{(\sum X)^2}{n} \right) \right] \left[ \sum Y^2 - \left( \frac{(\sum Y)^2}{n} \right) \right]}} = \frac{COV(X, Y)}{\sigma_x \sigma_y}$$

**ANOVA Table for Regression**

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	1	SSR	MSR = SSR/1
Error/Residual	$n - 2$	SSE	MSE = SSE/( $n - 2$ )
Total	$n - 1$	SST	

**Test Statistic for Significance of the Predictor Variable**

$$t_i = \frac{b_i}{S_{b_i}} \text{ and the critical value} = \pm t_{\alpha/2, (n-p-1)}$$

*Where  $p$  = number of predictor***G. INDEX NUMBERS**

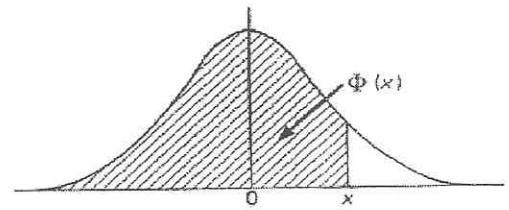
<b>Simple Price Index</b> $P = \frac{p_t}{p_0} \times 100$	<b>Laspeyres Quantity Index</b> $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
<b>Aggregate Price Index</b> $P = \frac{\sum p_t}{\sum p_0} (100)$	<b>Paasche Quantity Index</b> $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
<b>Laspeyres Price Index</b> $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	<b>Fisher's Ideal Price Index</b> $\sqrt{(\text{Laspeyres Price Index})(\text{Paasche Price Index})}$
<b>Paasche Price Index</b> $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	<b>Value Index</b> $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

## B. STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$ .  $\Phi(x)$  is

the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to  $x$ . When  $x < 0$  use  $\Phi(x) = 1 - \Phi(-x)$ , as the normal distribution with zero mean and unit variance is symmetric about zero.



$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
0.00	0.5000	0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452	2.00	0.97725
0.01	0.5040	0.41	0.6591	0.81	0.7910	1.21	0.8869	1.61	0.9463	2.01	0.97778
0.02	0.5080	0.42	0.6628	0.82	0.7939	1.22	0.8888	1.62	0.9474	2.02	0.97831
0.03	0.5120	0.43	0.6664	0.83	0.7967	1.23	0.8907	1.63	0.9484	2.03	0.97882
0.04	0.5160	0.44	0.6700	0.84	0.7995	1.24	0.8925	1.64	0.9495	2.04	0.97932
0.05	0.5199	0.45	0.6736	0.85	0.8023	1.25	0.8944	1.65	0.9505	2.05	0.97982
0.06	0.5239	0.46	0.6772	0.86	0.8051	1.26	0.8962	1.66	0.9515	2.06	0.98030
0.07	0.5279	0.47	0.6808	0.87	0.8078	1.27	0.8980	1.67	0.9525	2.07	0.98077
0.08	0.5319	0.48	0.6844	0.88	0.8106	1.28	0.8997	1.68	0.9535	2.08	0.98124
0.09	0.5359	0.49	0.6879	0.89	0.8133	1.29	0.9015	1.69	0.9545	2.09	0.98169
0.10	0.5398	0.50	0.6915	0.90	0.8159	1.30	0.9032	1.70	0.9554	2.10	0.98214
0.11	0.5438	0.51	0.6950	0.91	0.8186	1.31	0.9049	1.71	0.9564	2.11	0.98257
0.12	0.5478	0.52	0.6985	0.92	0.8212	1.32	0.9066	1.72	0.9573	2.12	0.98300
0.13	0.5517	0.53	0.7019	0.93	0.8238	1.33	0.9082	1.73	0.9582	2.13	0.98341
0.14	0.5557	0.54	0.7054	0.94	0.8264	1.34	0.9099	1.74	0.9591	2.14	0.98382
0.15	0.5596	0.55	0.7088	0.95	0.8289	1.35	0.9115	1.75	0.9599	2.15	0.98422
0.16	0.5636	0.56	0.7123	0.96	0.8315	1.36	0.9131	1.76	0.9608	2.16	0.98461
0.17	0.5675	0.57	0.7157	0.97	0.8340	1.37	0.9147	1.77	0.9616	2.17	0.98500
0.18	0.5714	0.58	0.7190	0.98	0.8365	1.38	0.9162	1.78	0.9625	2.18	0.98537
0.19	0.5753	0.59	0.7224	0.99	0.8389	1.39	0.9177	1.79	0.9633	2.19	0.98574
0.20	0.5793	0.60	0.7257	1.00	0.8413	1.40	0.9192	1.80	0.9641	2.20	0.98610
0.21	0.5832	0.61	0.7291	0.01	0.8438	1.41	0.9207	1.81	0.9649	2.21	0.98645
0.22	0.5871	0.62	0.7324	0.02	0.8461	1.42	0.9222	1.82	0.9656	2.22	0.98679
0.23	0.5910	0.63	0.7357	0.03	0.8485	1.43	0.9236	1.83	0.9664	2.23	0.98713
0.24	0.5948	0.64	0.7389	0.04	0.8508	1.44	0.9251	1.84	0.9671	2.24	0.98745
0.25	0.5987	0.65	0.7422	1.05	0.8531	1.45	0.9265	1.85	0.9678	2.25	0.98778
0.26	0.6026	0.66	0.7454	0.06	0.8554	1.46	0.9279	1.86	0.9686	2.26	0.98809
0.27	0.6064	0.67	0.7486	0.07	0.8577	1.47	0.9292	1.87	0.9693	2.27	0.98840
0.28	0.6103	0.68	0.7517	0.08	0.8599	1.48	0.9306	1.88	0.9699	2.28	0.98870
0.29	0.6141	0.69	0.7549	0.09	0.8621	1.49	0.9319	1.89	0.9706	2.29	0.98899
0.30	0.6179	0.70	0.7580	1.10	0.8643	1.50	0.9332	1.90	0.9713	2.30	0.98928
0.31	0.6217	0.71	0.7611	0.11	0.8665	1.51	0.9345	1.91	0.9719	2.31	0.98956
0.32	0.6255	0.72	0.7642	0.12	0.8686	1.52	0.9357	1.92	0.9726	2.32	0.98983
0.33	0.6293	0.73	0.7673	0.13	0.8708	1.53	0.9370	1.93	0.9732	2.33	0.99010
0.34	0.6331	0.74	0.7704	0.14	0.8729	1.54	0.9382	1.94	0.9738	2.34	0.99036
0.35	0.6368	0.75	0.7734	1.15	0.8749	1.55	0.9394	1.95	0.9744	2.35	0.99061
0.36	0.6406	0.76	0.7764	0.16	0.8770	1.56	0.9406	1.96	0.9750	2.36	0.99086
0.37	0.6443	0.77	0.7794	0.17	0.8790	1.57	0.9418	1.97	0.9756	2.37	0.99111
0.38	0.6480	0.78	0.7823	0.18	0.8810	1.58	0.9429	1.98	0.9761	2.38	0.99134
0.39	0.6517	0.79	0.7852	0.19	0.8830	1.59	0.9441	1.99	0.9767	2.39	0.99158
0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452	2.00	0.9772	2.40	0.99181

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
2.40	0.99180	2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918
41	99202	56	99477	71	99664	86	99788	01	99869	16	99921
42	99224	57	99492	72	99674	87	99795	02	99874	17	99924
43	99245	58	99506	73	99683	88	99801	03	99878	18	99926
44	99266	59	99520	74	99693	89	99807	04	99882	19	99929
2.45	0.99286	2.60	0.99534	2.75	0.99702	2.90	0.99813	3.05	0.99886	3.20	0.99931
46	99305	61	99547	76	99711	91	99819	06	99889	21	99934
47	99324	62	99560	77	99720	92	99825	07	99893	22	99936
48	99343	63	99573	78	99728	93	99831	08	99896	23	99938
49	99361	64	99585	79	99736	94	99836	09	99900	24	99940
2.50	0.99379	2.65	0.99598	2.80	0.99744	2.95	0.99841	3.10	0.99903	3.25	0.99942
51	99396	66	99609	81	99752	96	99846	11	99906	26	99944
52	99413	67	99621	82	99760	97	99851	12	99910	27	99946
53	99430	68	99632	83	99767	98	99856	13	99913	28	99948
54	99446	69	99643	84	99774	99	99861	14	99916	29	99950
2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918	3.30	0.99952

The critical table below gives on the left the range of values of  $x$  for which  $\Phi(x)$  takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of  $\Phi(x)$  indicated.

3.075	0.9990	3.263	0.9994	3.731	0.99990	3.916	0.99995
3.105	0.9990	3.320	0.9995	3.759	0.99991	3.976	0.99996
3.138	0.9991	3.389	0.9996	3.791	0.99992	4.055	0.99997
3.174	0.9992	3.480	0.9997	3.826	0.99993	4.173	0.99998
3.215	0.9993	3.615	0.9998	3.867	0.99994	4.417	1.00000
	0.9994		0.9999		0.99995		

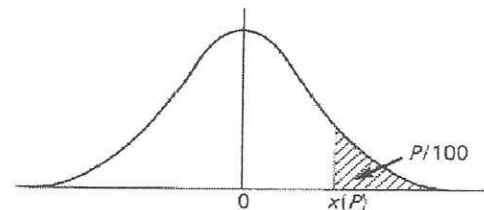
When  $x > 3.3$  the formula  $1 - \Phi(x) \approx \frac{e^{-x^2/2}}{x\sqrt{2\pi}} \left[ 1 - \frac{1}{x^2} + \frac{3}{x^4} - \frac{15}{x^6} + \frac{105}{x^8} \right]$  is very accurate, with relative error less than  $945/x^{10}$ .

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points  $x(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-t^2/2} dt.$$

If  $X$  is a variable, normally distributed with zero mean and unit variance,  $P/100$  is the probability that  $X \geq x(P)$ . The lower  $P$  per cent points are given by symmetry as  $-x(P)$ , and the probability that  $|X| \geq x(P)$  is  $2P/100$ .



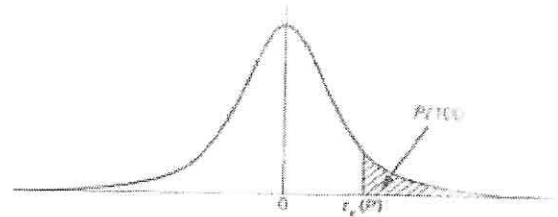
$P$	$x(P)$	$P$	$x(P)$	$P$	$x(P)$	$P$	$x(P)$	$P$	$x(P)$	$P$	$x(P)$
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.01	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.005	3.8906
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.001	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.0005	4.4172

TABLE 10. PERCENTAGE POINTS OF THE *t*-DISTRIBUTION

This table gives percentage points  $t_\nu(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}$$

Let  $X_1$  and  $X_2$  be independent random variables having a normal distribution with zero mean and unit variance and a  $\chi^2$ -distribution with  $\nu$  degrees of freedom respectively; then  $t = X_1/\sqrt{X_2/\nu}$  has Student's *t*-distribution with  $\nu$  degrees of freedom, and the probability that  $t \geq t_\nu(P)$  is  $P/100$ . The lower percentage points are given by symmetry as  $-t_\nu(P)$ , and the probability that  $|t| \geq t_\nu(P)$  is  $2P/100$ .



The limiting distribution of  $t$  as  $\nu$  tends to infinity is the normal distribution with zero mean and unit variance. When  $\nu$  is large interpolation in  $\nu$  should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.203	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.783	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.291	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.021	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291